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# MULTIMEDIA UNIVERSITY

## FINAL EXAMINATION

TRIMESTER 3, 2018/2019

**EEM2036 – ENGINEERING MATHEMATICS III**  
(ME/RE/TE)

1 JUNE 2019

9.00 A.M. – 11.00 A.M.

(2 Hours)

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### INSTRUCTIONS TO STUDENT

1. This exam paper consists of 8 **printed pages** (including cover page and formula sheets) with **four questions** only.
2. Attempt **ALL questions**. The distribution of marks for each question is given.
3. Please write all your answers in the Answer Booklet provided. Show all relevant steps to obtain maximum marks.
4. Only NON-PROGRAMMABLE calculator is allowed.

**Question 1**

a) Let  $A = \begin{bmatrix} -2 & -1 & 1 \\ 2 & 1 & 0 \\ 3 & 1 & -1 \end{bmatrix}$ . Find  $A^{-1}$ .

[5 marks]

b) Given that  $B = \begin{bmatrix} 2 & -3 \\ 1 & -2 \end{bmatrix}$ .

i) Find the eigenvalues of  $B$ . [2 marks]

ii) Prove that the eigenvectors for matrix  $B$  are  $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$  and  $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$  respectively.

[6 marks]

iii) Find the matrix  $P$  and the diagonal matrix  $D$  such that matrix  $B$  can be diagonalized via  $P^{-1}BP = D$ .

[4 marks]

iv) A particle moves in the plane in such a way that its position  $(x_1, x_2)$  at any time  $t$  satisfies the simultaneous differential equations

$$x_1' = 2x_1 - 3x_2$$

$$x_2' = x_1 - 2x_2$$

Using the solutions from (ii) and (iii) above, find the general solution for  $(x_1, x_2)$ .

[8 marks]

**Continued .....**

**Question 2**

- a) Find the volume,  $V$  of the solid in the first octant bounded by the coordinate planes and the graph of the equations  $z = x^2 + y^2 + 1$  and  $2x + y = 2$ .

[7 marks]

- b) Evaluate the integral  $\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2 + y^2} \, dy \, dx$  by changing to polar coordinate.

[9 marks]

- c) Use cylindrical coordinates to find the integral  $\int_0^1 \int_0^{\sqrt{1-y^2}} \int_0^{\sqrt{4-x^2-y^2}} z \, dz \, dx \, dy$ .

[9 marks]

**Continued ....**

**Question 3**

- a) If  $A = xz^3 i - 2x^2 yz j + 2yz^4 k$ , find  $\nabla \times A$  (curl  $A$ ) at the point  $(1, -1, 1)$ .

[5 marks]

- b) Find the directional derivative of  $\phi = x^2 yz + 4xz^2$  at  $(1, -2, -1)$  in the direction  $2i - j - 2k$ .

[7 marks]

- c) Suppose that a particle travels once around the unit circle in the counterclockwise direction and that at the point  $(x, y)$  it is subject to the force  $F(x, y) = (e^x - y^3)i + (x^3 + \cos y)j$ . Use Green's theorem to find the work done by the force.

[6 marks]

- d) Use Green's Theorem to evaluate the integral  $\int_C \ln(1+y) dx - \frac{xy}{1+y} dy$ , where  $C$  is the triangle with vertices  $(0, 0)$ ,  $(2, 0)$  and  $(0, 4)$ .

[7 marks]

**Continued ....**

**Question 4**

- a) Use composite Simpson's rule to estimate  $\int_0^2 x e^x dx$  with  $n = 4$ .

Subsequently, find the absolute error.

[13 marks]

- b) Given  $\frac{dy}{dx} = 3y + 2x$ , where  $y(0) = 1$  and  $h = 0.2$ .

Approximate the solution for the differential equation for one iteration only by using Runge Kutta method of order two.

[8 marks]

- c) Set up the Gauss-Siedel iterative equations the following linear system:

$$6x_1 - 3x_2 = 2$$

$$-x_1 + 3x_2 + x_3 = 1$$

$$x_2 + 4x_3 = 3$$

(Do not solve)

[4 marks]

**Continued ....**

## APPENDIX

TABLE OF FORMULAS

1. The  $n$ th Lagrange interpolating polynomial (LIP)

$$f(x) \approx P_n(x) = \sum_{i=0}^n L_i(x) f(x_i)$$

with

$$L_k(x) = \prod_{\substack{i=0 \\ i \neq k}}^n \frac{(x - x_i)}{(x_k - x_i)}.$$

2. Newton's divided-difference interpolating polynomial (NDDIP)

$$P_n(x) = f[x_0] + \sum_{k=1}^n f[x_0, x_1, \dots, x_k](x - x_0) \cdots (x - x_{k-1}).$$

3. The error in interpolating polynomial.

$$f(x) - P_n(x) = \frac{(x - x_0)(x - x_1) \cdots (x - x_n)}{(n+1)!} f^{(n+1)}(c_x)$$

for each  $x \in [x_0, x_n]$ , a number  $c_x \in (x_0, x_n)$  exists.

4. Newton's forward-difference formula

$$P_n(x) = f[x_0] + \sum_{k=1}^n \binom{s}{k} \Delta^k f(x_0).$$

5. Newton's backward-difference formula

$$P_n(x) = f[x_n] + \sum_{k=1}^n (-1)^k \binom{-s}{k} \nabla^k f(x_n).$$

6. Forward difference formula

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}.$$

Backward difference formula

$$f'(x) \approx \frac{f(x) - f(x-h)}{h}.$$

The error term for both forward and backward difference formula is

$$\left| \frac{h}{2} f''(c_x) \right|.$$

**Continued...**

7. Central difference formula

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$$

with the error term

$$\left| \frac{h^2}{6} f^{(3)}(c_x) \right|$$

8. Trapezoidal rule

$$\int_a^b f(x) dx \approx \frac{h}{2} [f(a) + f(b)]$$

for some  $\xi$  in  $(a, b)$  and  $h = b - a$ , with the error term is  $\left| \frac{h^3 f''(\xi)}{12} \right|$ .

9. Composite Trapezoidal rule

$$\int_a^b f(x) dx \approx \frac{h}{2} \left[ f(a) + f(b) + 2 \sum_{j=1}^{n-1} f(x_j) \right]$$

for some  $\xi$  in  $(a, b)$  and  $h = \frac{b-a}{n}$ , with the error term is  $\left| \frac{(b-a)h^2 f''(\xi)}{12} \right|$ .

10. Simpson's rule

$$\int_a^b f(x) dx \approx \frac{h}{3} \left[ f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right]$$

for some  $\xi$  in  $(a, b)$  and  $h = \frac{b-a}{2}$ , with the error term  $\left| \frac{h^5 f^{(4)}(\xi)}{90} \right|$ .

11. Composite Simpson's rule

$$\int_a^b f(x) dx \approx \frac{h}{3} \left[ f(a) + 2 \sum_{j=1}^{\left(\frac{n}{2}\right)-1} f(x_{2j}) + 4 \sum_{j=1}^{\frac{n}{2}} f(x_{2j-1}) + f(b) \right]$$

for some  $\xi$  in  $(a, b)$  and  $h = \frac{b-a}{n}$ , with the error term  $\left| \frac{(b-a)h^4 f^{(4)}(\xi)}{180} \right|$ .

**Continued ...**

12. Newton-Raphson's method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 0, 1, 2, \dots$$

13. Euler's method

$$y_{i+1} = y_i + hf(x_i, y_i)$$

with local error  $\frac{h^2}{2} Y''(\xi_i)$  for some  $\xi_i$  in  $(x_i, x_{i+1})$ .

14. Runge Kutta method of order two (Improved Euler method)

$$y_{i+1} = y_i + \frac{1}{2}(k_1 + k_2)$$

$$k_1 = hf(x_i, y_i)$$

$$k_2 = hf(x_i + h, y_i + k_1)$$

15. Runge Kutta method of order four

$$k_1 = hf(x_i, y_i),$$

$$k_2 = hf(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1),$$

$$k_3 = hf(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_2),$$

$$k_4 = hf(x_{i+1}, y_i + k_3),$$

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4).$$

**End of Paper**